Null Ronchi-Hartmann test for a lens

Maximino Avendaño-Alejo,1,* Dulce González-Utrera,1 Naser Qureshi,1 Luis Castañeda,2 and César L. Ordóñez-Romero3

1 Universidad Nacional Autónoma de México, Centro de Ciencias Aplicadas y Desarrollo Tecnológico, Apdo. Postal 70-186, C. P. 04510, Distrito Federal, México
2 Benemérita Universidad Autónoma de Puebla, Instituto de Física, Apdo. Postal J-48, C. P. 72570, Puebla, México
3 Universidad Nacional Autónoma de México, Instituto de Física, Circuito de la Investigación Científica C. P. 04510, Distrito Federal, México

*maximino.avendano@ccadet.unam.mx

Abstract: A method to design Ronchi-Hartmann screens for improved alignment in the testing of fast plano-convex spherical lenses is presented. We design null screens that produce aligned straight fringes for observed patterns. The designs of these null screens are based on knowledge of the caustic by refraction. A qualitative test for a lens is presented.

© 2010 Optical Society of America

OCIS codes: (120.5710) Refraction; (220.1140) Alignment; (220.4840) Testing.

References and links


Hartmann and Ronchi tests have been a topic of research for many years. In these tests, sampling screens with either holes or fringes with varying separations are used to test lenses and mirror surfaces [1–5]. The Hartmann test traditionally uses a screen with a uniform distribution of holes, which produces a non-uniform distribution of bright spots at the detection plane either
by reflection or refraction. For fast optical surfaces, this non-uniformity can be very pronounced i.e., \( F/\# < 1 \), complicating the subsequent analysis. To overcome this problem a null test has been developed for aspherical convex surfaces \([6]\) and off-axis mirrors \([7, 8]\), in which instead of using an array of point light sources, null screens printed with a laser printer are used to mask a uniform light source. Null tests are motivated by the fact that the interpretation of the surface shape is simplified, and the visual analysis turns out to be straightforward for both qualitative and quantitative tests.

In earlier papers, special rulings with curved lines were constructed to test aspherical concave mirrors and this approach was denominated the null Ronchi test. In Ref. \([4]\) the ruling was computed according to the following procedure: using a ray tracing program, a transverse aberration curve (\( TA \)) is computed at the approximate place where the ruling is to be placed. A system of five linear simultaneous equations is solved in order to determine the coefficients of the aberration polynomial \( TA \). Finally, using geometrical considerations and the computed \( TA \) the null screens are designed. Alternatively, in Ref. \([5]\), a predetermined formula is used for the \( TA \) and with the aid of a formula to compensate for the error introduced by the Ronchigram fringes over the mirror surface, and going through geometrical considerations, a simple formula is obtained and solved using Newton’s method to design null screens. At the present time it is possible to design null screens using lens design programs \([9, 10]\).

In this paper we design a null screen that produces a uniform distribution of straight fringes in which both dark and bright areas have the same width at the detection plane. Due to the position at which the null screen is placed, the procedure can be considered a Hartmann test instead of a Ronchi test, although it has been shown in earlier papers that both tests are equivalent \([11]\). We therefore simply call this a null Ronchi-Hartmann test. The design of this screen is based on knowledge of the caustic surface by refraction or diacaustic, and does not require the aberration polynomial \( TA \). As is well known, the testing of an optical system is best accomplished if no additional optics are needed. An important feature of this paper is that we use only a CCD sensor at the detection plane without additional optics.

1. Theory

We define the \( Z \) axis to be parallel to the optical axis, we assume that the \( Y - Z \) plane is the plane of incidence, which is a cross section of a spherical refractor of radius \( R \) centered at \( \mathcal{E} \) and we define the origin of the system to be at the vertex of the lens \( \mathcal{E} \). We assume that there is rotational symmetry about \( Z \) axis. Without loss of generality we assume that light rays enter from the left and that the point source is placed at infinity. We consider ideally a bundle of rays crossing the plane face of the lens without being deflected and propagating towards the spherical surface as is shown in Fig. 1(a). We can write the caustic equation produced by a plano-convex spherical lens according to Ref. \([12]\) as

\[
(z(h), y(h)) = \left( n_t \left[ n_t n_a^2 (R^2 - h^2)^{3/2} + (R^2 n_a^2 - h^2 n_t^2)^{3/2} \right] \frac{n_t}{n_a^2 (n_t^2 - n_a^2) R^2} - R, \frac{h^3 n_t^2}{R^2 n_a^2} \right), \tag{1}
\]

which gives the coordinates of the locus of points that parametrically represent the caustic produced by a lens, where \( h \) is the height parameter for each incident ray, \( R \) is the radius, \( t \) is the thickness and \( n_t \) is the index of refraction of the lens, and \( n_a \) is the index of refraction of air as shown in Fig. 1(a).

Throughout this paper we use the following parameters for the lens: \( R = 38.76 \text{mm}, t = 32.68 \text{mm}, n_a = 1 \) and \( n_t = 1.517 \) for \( \lambda = 633 \text{nm} \), its diameter is \( D = 75 \text{mm} \), with \( EFL = 75 \text{mm} \). We also use a CCD sensor whose major and minor lengths are \( L_M = 8.8 \text{mm} \) and \( l_m = 6.6 \text{mm} \) respectively, with resolution \( 640 \times 480 \) pixels. The light source used is a He-Ne laser \( (\lambda =\)
Fig. 1. (a) Caustic produced by refraction through a Plano-Convex spherical lens and the associated parameters considering that the point source is located at infinity. (b) Diagram of the experimental setup to test a plano-convex spherical lens using null screens.

633nm). The diagram of the experimental setup is shown in Fig. 1(b), where we can also see two polarizers which have been used to reduce the intensity of the laser beam.

2. Design of ruling for the null screen

To calculate the heights $h$ of the points on the Ronchi-Hartmann null screen that yield bands of equal width along a predefined side of the CCD sensor (i.e. at the detection plane), we proceed backwards, starting at the detection plane. The $z_s$ and $y_s$ coordinates of the point of incidence of the ray on the lens are not selected a priori but must be calculated as is traditionally done: given an arbitrary point $P_0 = (z_0, y_0)$ at the detection plane, we trace a ray back through a point whose coordinates are $P_i = (z_i, y_i)$, in order to provide the equation for a straight line, and finally, this line is extended to reach the lens at $P_s = (z_s, y_s)$ as is shown in Fig. 2(a). It is well known, the caustic surface is the envelope of the family of refracted rays produced by a point source. In this way, all refracted rays are tangent to the caustic surface. A point $P_i$ lies on the caustic surface whose coordinates are given by Eq. (1) and with the aid of the point $P_0$ it is possible to provide an equation for an arbitrary tangent to the caustic surface given by

$$\tan \psi = \frac{y_0 - h^3 n_i^2}{z_0 + R - n_l [n_i n_0^2 (R^2 - h^2)^{3/2} + (R^2 n_i^2 - h^2 n_l^2)^{3/2}]}.$$  \hspace{1cm} (2)

Furthermore, we know that the derivative with respect to $h$ from Eq. (1) evaluated at the point $P_i$ also gives the tangent to the caustic surface. We thus obtain

$$\tan \psi = \frac{(n_i^2 - n_l^2) h}{n_i^2 \sqrt{R^2 - h^2} + n_l \sqrt{n_l^2 R^2 - n_l^2 h^2}}.$$  \hspace{1cm} (3)

By equating the set of Eqs. (2) and (3) we obtain a polynomial equation for $h$ given by

$$h \{n_0^2 (n_i^2 - n_0^2)(z_0 + R)R^2 - n_l [n_l n_0^2 (R^2 - h^2)^{3/2} + (R^2 n_l^2 - h^2 n_l^2)^{3/2} \} = (h^3 n_i^2 - y_0 R^2 n_l^2) \left( n_0^2 \sqrt{R^2 - h^2} + n_l \sqrt{n_l^2 R^2 - n_l^2 h^2} \right).$$  \hspace{1cm} (4)

In this way, by solving the above for $h$ through numerical methods we obtain the heights $h_s$ (the subscript $s$ means screen), exclusively for a point on the Y-axis, which will form part of
the null screen as shown in Fig. 2(a). To design the null screen in such a way that its image formed by refraction yields a uniform array of bright fringes at the image plane, we define a point \((z_0, y_0)\) placed on a straight line, although it could also be placed at any curve. In this way, \(z_0\) is the distance along the optical axis where the detection plane is placed. In order to provide a planar screen, we define \(y_0 = (a_i^2 + a_j^2)^{1/2}\), in such a way that \(a_i = (1/2 + i)\Delta,\) for \(i = 0, 1, \ldots, n\), where \(n\) is the number of lines which will form the fringes, whose separation is given by \(\Delta = L_M/(1 + 2n)\). We have also defined \(a_j = j\Delta,\) for \(j = 0, 1, \ldots, m\), where \(m\) is the number of points which form a continuous line for each fringe whose separation between contiguous points is given by \(\Delta_y = l_m/(2m)\).

Due to symmetry it is enough to consider only one quarter of the CCD sensor and we choose the first quadrant given by \((a_i \geq 0, a_j \geq 0)\) coordinates, or in other words, \(a_i \in [\Delta_x/2, L_M/2]\) and \(a_j \in [0, l_m/2]\) as is shown in Fig. 2(b). It is important to comment that we must choose the correct solution for each \(h_s\) in order to design a suitable null screen for a lens under test. In this way we demand that \(h_s\) is a real number, obtaining two cases for the positioning of \(z_0:\) For \(z_0 \geq EFL\), we have a unique solution whose values lie in the range \([0, D/2]\) for \(y_0 < 0\) and \([-D/2, 0]\) for \(y_0 > 0\), where \(D\) is the diameter of the lens. For \(z_0 < EFL\) the plane of detection lies inside the caustic surface. In this particular case there are three possible solutions because we have rays from the lower part and rays arriving from the upper part of the lens which coincide in \(y_0\) when the plane of detection is placed inside of the caustic region, as shown in Fig. 2(a). Further research will extend these results either to design null screens or to produce an interferometer inside the caustic region. Finally, knowing the values for \(h_s\), the positions of the points on the lines which form the null screen are given by

\[
(x_s, y_s) = (h_s \cos \phi_{ij}, h_s \sin \phi_{ij}),
\]

where \(\phi_{ij} = \arctan(a_j/a_i)\) for \(a_i \neq 0\) as is shown in Fig. 2(b).

3. Null screens: linear ruling

An important consideration is that if visual observations are to be made in real time using the experimental setup, the computed ruling used as null screen can be calculated to produce an observable pattern of adequate size for the plano-convex spherical lens under test at a unique detection plane. On the other hand, if we desire to observe a pattern with a predetermined number of fringes and separations between them, the fringes should be designed in such a way that they completely fill the area of the CCD.
By substituting the values given above for \( n_i, n_a \) and \( R \) into Eq. (4), where we have considered \( n = 15 \) lines with \( m = 50 \) points for each line and that the CCD sensor is placed at \( z_0 = 1.0EFL \), we obtain the points \( h_i \), whose values are introduced into Eq. (5). Finally, rotating these points about the \( X \)-axis and about the \( Y \)-axis we obtain the lines which form the null screen as shown in Fig. 3(a). To produce the null screen we select two contiguous lines and put them together in order to close a continuous curve in such a way that we can illuminate it alternatively as is shown in Fig. 3(b). The null screen is printed on an acetate foil using a traditional laser printer specified at 1200 dpi. After placing the null screen according to Fig. 1(b) the image recorded on the CCD sensor is shown in Fig. 3(c), where we can see an array of fringes with a non-uniform intensity distribution. Furthermore, on the left side there are, as expected, fringes that have a slight curvature as opposed to straight fringes due to slight deformations of the lens under test. It is important to say that in Refs. [6–8] an extended light source and a positive lens is used to focus light on CCD sensor. In this proposal, since the laser beam that passes through the lens under test is directly incident on CCD sensor, we suggest that the intensity recorded at the center is higher than on the sides of the CCD due to Fresnel’s losses inside the lens. This is shown in Fig. 3(c). We have used a collimator lens with \( F/# = 6 \), in order to produce a uniformly collimated beam. In other words, the central hot spot has been reduced by increasing the distance between the collimator lens and the pinhole.

The non-uniformity of intensity recorded on the CCD sensor could be produced by several factors. Primarily: (I) To construct a plane wave we create a point source by using a pinhole. This point source emits a spherical wave which is converted into a plane wave using a lens. Unfortunately, a point source is an unattainable ideal and there is an inevitable non-isotropy of light exiting the pinhole. (II) Effects of printing the null screens on acetate foil: we are assuming that the acetate foil is a perfect plane-parallel sheet and we are assuming that this foil is free of strain and stress after the printing process. A graduated filter at the collimator, or even as part of the null screen, can be used as a possible solution to obtain a uniform intensity distribution of the incident beam on the CCD sensor.

The test is very sensitive to alignment and positioning. For example in Fig. 4(a) the CCD sensor is placed inside the caustic region, and the recorded image resembles a slightly decentered pincushion aberration. In other words, the null screen is laterally displaced along the \( X \)-axis. In Fig. 4(b) the null screen is centered, which means that it is aligned with the optical axis and it also lies inside the caustic region. In Fig. 4(c) a zoom of the center of Fig. 4(b) by reducing the intensity of laser beam is presented, where we can see clearly that the straight lines change to symmetrical loops inside the caustic region if the experimental setup is aligned, and the loops are asymmetrical when it is not centered. A theoretical screen having similar features
has been reported in Ref. [7]. Finally in Fig. 4(d) the CCD sensor is placed near the circle of least confusion [12], and the loops are symmetric along the X-axis.

![Image](a) The CCD sensor is decentered and lies inside the caustic. (b) The CCD sensor is centered and lies inside the caustic. (c) Zoom of the center of figure b by reducing the intensity of the laser beam. (d) CCD sensor is placed near of the circle of least confusion.

Fig. 4. An error analysis and a quantitative test for measuring the index of refraction of fast and slow plano-convex lenses using this method has been well explained in Ref. [13]. The central idea of this paper is to extend this result to obtain the shape of the lens under test by using null screens through the traditional procedure of the Ronchi test. The Ronchi and Hartmann test are considered as geometric methods, so that this test is in the geometric optics regime. We have designed a null screen in such a way that its image, which is formed by refraction passing through the lens under test, becomes either a linear array of fringes or an exact square array if the surface is perfect, and any departure from this geometry is indicative of defects on the surface. Here the surface can be tested in a single measurement. The surface departures from the best surface fit are of the order of 1 μm when the errors in the determination of the coordinates of the centroids of the refracted images are less than 1 pixel, and the errors in the coordinates of the spots of the null screen are less than 0.5mm, according to Ref. [8]. An error analysis including the numerical precision for each $h_i$ and their positioning after the printing on the acetate foil are necessary in order to improve the precision of the null Ronchi-Hartmann test for a quantitative evaluation.

4. Null screens: square ruling

As was explained in Ref. [5], to obtain a distribution of holes on a screen that produces a regular grid in the observation plane in the form of a square array, the positions of the centers of the holes on the Hartmann screen must be on the crossing points of two superimposed computed Ronchigrams rotated 90° with respect to each other, it is shown in Fig. 5(a), where the vertical curves are thicker than the horizontal lines because the null screen is design to fill the rectangular area of the CCD sensor. The image is recorded on the CCD sensor using the null screen as is shown in Fig. 5(b). We can see a linear array, which presents distortion and defocusing at the corners. This effect is due to slight deformations of the lens under test as was explained above, and we will use the procedure of the traditional Ronchi test to obtain the shape of the lens under test in future works. A zoom of the center of an image recorded by reducing the laser beam intensity when the CCD sensor is placed inside the caustic is shown in Fig. 5(c); this image is symmetric but the intensity is slightly non-uniform. The screen image for a plane that is further away from the surface than the design distance is shown in Fig. 5(d).

Conclusions

A simple method to design null screens by refraction through a plano-convex lens has been presented and either Ronchi or Hartmann null screens can be designed by using this method.
Fig. 5. (a) Design of the null Hartmann screen. (b) Image recorded at the design plane. (c) Zoom of the center for a null screen inside the caustic by reducing the intensity of the laser beam. (d) Zoom of the center for a null screen when it is further away from the surface than the design distance.

The test is very sensitive to positioning and alignment of optical systems, as we can see from the images inside the caustic region. Although the polynomial equation [Eq. (4)] can be solved analytically it is enough to solve it through numerical methods. This method opens the door to designing null screens to test either conic or aspherical plano-convex lenses either outside or inside of the caustic region.

Acknowledgments

This work was partially supported by PAPIIT-UNAM project numbers #IN113510 and IN114909 and by CONACyT project number #82829. The corresponding author is grateful to I. Goméz-García and the anonymous reviewers for her valuable assistance and invaluable comments.