Huygens’ Principle: Exact wavefronts produced by aspheric lenses

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Abstract: We obtain simple exact formulas for the refracted wavefronts through plano-convex aspheric lenses with arbitrary aspheric terms by considering an incident plane wavefront propagating along the optical axis. We provide formulas for the zero-distance phase front using the Huygens’ Principle and the Malus-Dupin theorem. Using the fact that they are equivalent, we have in the second method found a way to use an improper integral, instead of the usual evaluated integral, to arrive at these formulas. As expected, when the condition of total internal reflection is satisfied, there is no contribution to the formation of the refracted wavefront.

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References and links

1. **Introduction**

The aspheric lens can help simplify optical system design by minimizing the number of elements required and yields sharper images than conventional lenses. Aspherical elements are particularly useful for correcting distortion in wide-angle lenses. In summary aspheric optical surfaces deliver higher performing, more compact, and lighter systems in a wide range of applications. There are three ways to make an aspherical lens. The basic way is to grind a piece of glass down to the right shape. This is quite difficult due to the extreme precision required to achieve the complex geometry. Another way is to mould a glass lens element. Finally one can cement a plastic resin aspheric surface to the top of a glass spherical element [1].

The caustic can be defined as the locus of the principal centers of curvature of a wavefront [2]. In other words, if equal optical paths are measured along each ray from the source, the surface constructed by the end points will be normal to all the rays. These surfaces are the phase fronts of the wave system, for which the rays are defined within the geometrical optics approximation. Although the caustics and wavefronts either by reflection or refraction are part of a well known subject [3–5], the contribution in this work it is to provide simple formulas of the wavefront surfaces caused by refraction on plano-convex aspheric lenses exclusively in a meridional plane by using the Huygens’ principle and the theorem of Malus-Dupin. In this manuscript we consider the aspheric equation given according to [5], although recently there have been defined new formulas to represent this kind of surface [6, 7].

The treatment in this work is physically equivalent to that presented by Shealy and Hoffnagle [3, 4] in their solution to the wavefront of an aspheric lens, and the results presented here illustrate the equivalence of the Huygens’ principle and the eikonal treatment. On a practical level, this Huygens’ approach has advantages: since the treatment here involves performing a sum of wavelets whose envelope will form the wavefront and does not imply solving a differential equation, we do not need to construct initial boundary conditions. The final result is a relatively simple solution that can be used to directly compute the wavefront produced by an arbitrary aspheric lens, and vice versa, starting with the aspheric coefficients used in practical applications. From an optical design point of view, this is applicable to aspheric lens design and more generally to the design of arbitrary lenses for waveform shaping.

2. **Huygens’ Principle and its phase fronts.**

Throughout this manuscript lower cases for \((z, y)\) will be used to designate either formulas for the caustic surfaces or refracted rays through lenses and upper cases for \((Z, Y)\) will be used to designate the geometrical wavefront. So, we define that the \(Z\) axis is parallel to the optical axis, we assume that the \(Y – Z\)-plane is the plane of incidence, which is a cross section of a plano-convex aspheric lens with an arbitrary number of aspheric coefficients, whose paraxial radius is \(R\), and the origin of the system \(O\) is placed at the vertex of the plano-convex lens. We assume that a plane wave is incident on the lens parallel to the optical axis, crossing the plane face of the lens without being deflected, and this is propagated to the aspheric surface. In this way the Huygens’ principle assumes that a wavefront is the envelope of an aggregate of wavelets centered on a previous wave front in the wave-front train. The idea here is to find the family of wavelets that are centered on the aspheric refracting surface and whose envelope yields the refracted wavefront, which also is called the phase front, zero-distance phase front or alternatively the archetype wavefront. We follow a procedure similar to [8], in such a way that it is easy to see that the wavelets are formed by a disturbance of circles centered along the...
aspheric surface with radii varying as are shown in Fig. 1(a), and they are represented by

\[(Z - [t + S_h])^2 + (Y - h)^2 = \left(\frac{n_i}{n_a} S_h\right)^2, \quad h \in [-H, H]\]  \hspace{1cm} (1)

where \(H\) is the entrance aperture, \(t\) is the thickness of the lens, \(n_i\) the index of refraction of the lens for a predefined wavelength which is immersed in a medium with index of refraction \(n_a\), \((n_i > n_a)\), and where we have assumed that \(S_h\) represents the aspheric equation given by

\[S_h = \frac{c h^2}{1 + \sqrt{1 - (k + 1)c^2 h^2}} + \sum_{i=2}^{N} A_{2i} h^{2i},\]  \hspace{1cm} (2)

where \(c = 1/R\) is the paraxial curvature, \(k\) is the conic constant, \(A_4, A_6, A_8, \ldots A_{2N}\), are the aspheric order terms with \(N\) arbitrary and \(h\) represents either the height for an arbitrary incident ray or the height for the centers of the wavelets along \(Y\)-axis lying on the meridional plane. It is worth commenting that we are using the sign convention suggested in [9]. In order to obtain the wavefront, which is the envelope of the wavelets represented by Eq. (1), we differentiate Eq. (1) with respect to \(h\) and reducing further we have

\[(Y - h) = -\left[(Z - [t + S_h]) + \left(\frac{n_i}{n_a}\right)^2 S_h\right] S'_h,\]  \hspace{1cm} (3)

where \(S'_h\) is the first derivative with respect to \(h\), thus from Eq. (2) we obtain

\[S'_h = \frac{ch}{\sqrt{1 - (k + 1)c^2 h^2}} + \sum_{i=2}^{N} 2iA_{2i} h^{2i-1}.\]  \hspace{1cm} (4)

Finally, solving for \(Z\) and \(Y\) by using Eqs. (1) and (3) we get

\[Z_{\pm} = t + \frac{S_h \sqrt{n_a^2 + (n_a^2 - n_i^2) S'_h^2}}{\sqrt{n_a^2 + (n_a^2 - n_i^2) S'_h^2 \pm n_i}},\]  \hspace{1cm} (5)

\[Y_{\pm} = h - \frac{n_i S_h S'_h}{n_a^2 (1 + S'_h^2)} \left[ n_i \pm \frac{n_a^2 + (n_a^2 - n_i^2) S'_h^2}{n_a^2} \right].\]
where the subscript (+) sign provides the equation of a wavefront progressing in a retrograde direction and is ignored, and for (−) sign it yields a wavefront progressing in the forward direction as shown in Fig. 1(b), both wavefronts are tangent to each of the wavelets centered along the aspheric surface passing through the vertex V of the lens. Note that the continuous circles in the figures indicate the envelope of the forward propagating wave and the dotted circles form the envelope of a backward propagating wave that is a nonphysical solution and is not taken into account in the subsequent analysis. We consider exclusively from Eq. (5) a wavefront progressing in the forward direction and reducing further it becomes

\[
(Z_{pc}, Y_{pc}) = \left( t + \frac{(n_a^2 - n_i^2) S_h \sqrt{n_a^2 + (n_a^2 - n_i^2) S_h^2}}{n_a^2[n_i + \sqrt{n_a^2 + (n_a^2 - n_i^2) S_h^2}]}, h + \frac{n_i(n_a^2 - n_i^2) S_h S_h'}{n_a^2[n_i + \sqrt{n_a^2 + (n_a^2 - n_i^2) S_h^2}]}, \right),
\]

where the subscript \( pc \) means plano-convex. It is important to say that Eq. (6) gives the coordinates of the locus of points that parametrically represent the zero-distance phase front or the archetype wavefront, which is produced by a plano-convex aspheric lens in a meridional plane as a function of \( h \) for a plane wavefront incident on the lens. From Eq. (6) assuming that the radical \( n_a^2 + (n_a^2 - n_i^2) S_h^2 = 0 \), and solving for \( h \) whose solution is defined by \( h_c \), the rays undergo Total Internal Reflection (TIR), see for example [5]. Physically, if \( h_c > H \), then all the wavelets contribute to the formation of the refracted wavefront, being tangent to the wavefront as is shown in Fig. 2(a). On the other hand, if \( h_c < H \) there are wavelets which do not contribute to the formation of the wavefront, and therefore they are not tangent to the wavefront as is shown in Fig. 2(b).

Fig. 2. Wavelets and wavefronts produced by a plano-convex aspheric lens, considering a plane wavefront incident on the lens. (a) Without TIR. (b) Showing TIR.

3. An alternative method

A congruence is formed by rays propagating in an homogeneous isotropic medium, which originate from a point source being the congruence perpendicular to the phase front. The theorem of Malus and Dupin states that the propagating phase front in a uniform medium, is everywhere
normal to the congruence of rays [10]. Since the caustic is the locus of the principal centers of curvature of a wavefront, also the caustic is the evolute of the zero-distance phase front, in others words the phase front is the involute of the caustic. Mathematically, if a curve $\mathcal{A}$ is the involute of curve $\mathcal{B}$, then $\mathcal{B}$ is the evolute of $\mathcal{A}$, and vice versa. Let $\mathcal{C}$ be a curve in parametric form $\mathcal{C} = [f(h), g(h)]$, in such a way that the equation of the involute is given by

$$[F, G] = [f, g] - \left[ \frac{\partial}{\partial h} \left[ \frac{f}{g} \right] \frac{\partial}{\partial h} \right] \int dh \sqrt{\left( \frac{\partial f}{\partial h} \right)^2 + \left( \frac{\partial g}{\partial h} \right)^2},$$

(7)

where for simplicity the constant of integration has been considered null. According to Eq. (7) in reference [5], the caustic surface $(z_{pc}, y_{pc})$ for plano-convex aspheric lenses was written as

$$z_{pc} = t + Sh + \frac{n_a^2 + (n_a^2 - n_i^2)S_h^2}{n_a^2 + n_i \sqrt{n_a^2 + (n_a^2 - n_i^2)S_h^2}} \left[ n_a^2(n_a^2 - n_i^2)S_h' \right],$$

(8)

$$y_{pc} = h - \frac{n_a^2 + (n_a^2 - n_i^2)S_h^2}{n_a^2 S_h'},$$

where $S_h''$ is the second derivative from Eq. (2). It is important to say that Eq. (8) gives the coordinates of the locus of points that parametrically represents the envelope of the family of refracted rays produced by an aspheric lens in a meridional plane as a function of $h$ when the point source is placed at infinity. Additionally, Eq. (8) is similar to Eq. (29a) for $C_+$ which provides the radii of curvature of the wavefront along the tangential plane according to reference [3], by using the follows considerations: $r = h$, $z(r) = S_h$, $n_1 = n_i$, $n_2 = n_a$ and $t = 0$. By substituting Eq. (8) into Eq. (7) for $z_{pc} \rightarrow f$ and $y_{pc} \rightarrow g$ and reducing further we obtain Eq. (6).

It is worth stating that the constant of integration omitted in Eq. (7) simply translates the vertex of the zero-distance phase front along the optical axis.

4. Exact wavefront for plano-convex aspheric lenses

Since either the zero-distance phase front or the archetype wavefront can also be considered as a source of such a congruence, all ensuing phase fronts are parallel in the geometrical sense. That is, points on each phase surface are equidistant where the distances are measured along the common normal between points on each of the phase front surfaces, according to Huygens’ principle as are shown in Fig. 3(a). Let $W_0$ be a point on a phase front whose profile is given parametrically by $W_0 = [F(h), G(h)]$, then $W$ will be on a parallel phase front at a distance $\mathcal{L}$ according to the following equation

$$W = \left( \frac{F \pm \left[ \frac{\partial G}{\partial h} \right] \mathcal{L}}{\sqrt{\left( \frac{\partial F}{\partial h} \right)^2 + \left( \frac{\partial G}{\partial h} \right)^2}}, \frac{G \pm \left[ \frac{\partial F}{\partial h} \right] \mathcal{L}}{\sqrt{\left( \frac{\partial F}{\partial h} \right)^2 + \left( \frac{\partial G}{\partial h} \right)^2}} \right).$$

(9)

In this way, Eq. (9) for $W_{(-,+)}$ provides a retrograde wavefront and for $W_{(+,-)}$ provides the forward wavefront. Substituting Eq. (6) into Eq. (9) for $Z_{pc} \rightarrow F$ and $Y_{pc} \rightarrow G$ and reducing further we get

$$Z_{||pc} = Z_{pc} + \frac{n_a^2 + n_i \sqrt{n_a^2 + n_i^2 S_h^2}}{n_i + \sqrt{n_a^2 + n_i^2 S_h^2}} \left[ \frac{\mathcal{L}}{n_a} \right],$$

(10)

$$Y_{||pc} = Y_{pc} - \frac{n_a^2 - S_h'}{n_i + \sqrt{n_a^2 + n_i^2 S_h^2}} \left[ \frac{\mathcal{L}}{n_a} \right].$$
where the subscript $\parallel_{pc}$ means wavefronts propagating in parallel form produced by plano-
convex lenses, where $(Z_{pc}, Y_{pc})$ are defined in Eq. (6). We can see that for $L = 0$, Eq. (10) is
reduced to Eq. (6) providing the zero-distance phase front or archetype wavefront as expected,
and is shown in Fig. 3(b).

5. Wavefront for aspheric convex-plano lenses

Let $H$ be the height for the marginal wavelet, so that the family of wavelets centered along the
first aspheric surface of the lens can be represented by the following formula

$$(Z - S_h)^2 + (Y - h)^2 = \left( \frac{n_a}{n_i} (S_H - S_h) \right)^2, \quad h \in [-H, H]$$

(11)

where $S_H$ is evaluated at $H$ from Eq. (2), considering in this configuration that $c > 0$. For
example from Eq. (11), for $h = Y = 0$, then $S_h = 0$ and therefore it reduces to: $n_iZ = n_aS_H$
as is shown in Fig. 4(a). Following all the steps explained above to obtain the envelopes to the
wavelets of Eq. (11), these can be represented by

$$Z \pm = S_H - \frac{(S_H - S_h) \sqrt{n_i^2 + (n_i^2 - n_a^2) S_h^2} \left[ \sqrt{n_i^2 + (n_i^2 - n_a^2) S_h^2} \pm n_a \right]}{n_i^2 \left( 1 + S_h^2 \right)},$$

$$Y \pm = h + \frac{n_a (S_H - S_h) S_h}{n_i^2 \left( 1 + S_h^2 \right)} \left[ n_a \pm \sqrt{n_i^2 + (n_i^2 - n_a^2) S_h^2} \right],$$

(12)

as is shown in Fig. 4(b). Finally, from Eq. (12) we choose the wavefront progressing in the
forward direction $(Z_+, Y_+)$ inside the lens and reducing further it becomes

$$Z_{in} = S_H + \frac{(n_a^2 - n_i^2) (S_H - S_h) \sqrt{n_i^2 + (n_i^2 - n_a^2) S_h^2}}{n_i^2 \left( n_a \pm \sqrt{n_i^2 + (n_i^2 - n_a^2) S_h^2} \right)},$$

$$Y_{in} = h + \frac{n_a (n_a^2 - n_i^2) (S_H - S_h) S_h}{n_i^2 \left( n_a + \sqrt{n_i^2 + (n_i^2 - n_a^2) S_h^2} \right)},$$

(13)
It is simple to see that Eq. (13) represents the wavefront refracted inside the aspheric lens, in such a way that we can consider a new family of wavelets centered along the wavefront \((Z_{in}, Y_{in})\) and propagate it inside the lens up to the marginal wavelet reaching the plane face of the lens, propagating along the marginal ray instead of marginal wavelet touching the border of the lens at \((t, H)\) as is shown in Fig. 5(a). In other words, we obtain the length \(L_{H}\) between a point on aspheric surface \((S_{H}, H)\) and the plane face of the lens \((t, y_{i[H,t]}\) where \(y_{i[H,t]}\) represents the height of the marginal ray impinging on the plane face of the lens. By using the Snell’s law we provide the equation for the rays propagating inside the lens which can be written as

\[
y_{i | h, z} = h - \frac{(n_i^2 - n_a^2)(z - S_{h}) S'_{h}}{n_i^2 + n_a \sqrt{n_i^2 + (n_i^2 - n_a^2)S'^2_{h}}} \tag{14}
\]

where \(h\) and \(z\) are variables and we have considered that all the parameters of the lens are constants. Substituting \(h \rightarrow H\) and \(z \rightarrow t\) into Eq. (14), we calculate the distance along the refracted marginal ray according to \(L_{H} = \{(y_{i[H,t]} - H)^2 + (t - S_{H})^2\}^{1/2}\), and reducing further we get

\[
L_{H} = \frac{n_i(t - S_{H}) \left(n_a + \sqrt{n_i^2 + (n_i^2 - n_a^2)S'^2_{H}}\right)}{n_i^2 + n_a \sqrt{n_i^2 + (n_i^2 - n_a^2)S'^2_{H}}} \tag{15}
\]

\(S'_{H}\) is given from Eq. (4) evaluated at \(H\), we can see that \(L_{H}\) is constant and depends on all parameters involved in the process of refraction. By substituting Eq. (13) into Eq. (9) and reducing further we obtain a wavefront translated a distance \(L_{H}\) as is shown in Fig. 5(b), given by

\[
Z_{|in} = Z_{in} + \frac{n_i^2 + n_a \sqrt{n_i^2 + (n_i^2 - n_a^2)S'^2_{h}}}{n_i^2 + n_a \sqrt{n_i^2 + (n_i^2 - n_a^2)S'^2_{h}}} \frac{L_{H}}{n_i} \tag{16}
\]

\[
Y_{|in} = Y_{in} - \frac{(n_i^2 - n_a^2) S'_{h}}{n_i^2 + n_a \sqrt{n_i^2 + (n_i^2 - n_a^2)S'^2_{h}}} \frac{L_{H}}{n_i}\]
Finally, we can see that Eq. (16) provides the wavefront which will be refracted outside of the aspheric lens. Substituting \( h \rightarrow 0 \) into Eq. (16) provides the distance covered between the vertex \( \mathcal{O} \) of the lens and the translated wavefront along the optical axis, it becomes

\[
Z_{\parallel}[0] = \frac{n_a n_i t + \left[ n_i^2 t + (n_i^2 - n_a^2) S_H \right] \sqrt{n_i^2 + (n_i^2 - n_a^2) S_H^2}}{n_i \left( n_i^2 + n_a \sqrt{n_i^2 + (n_i^2 - n_a^2) S_H^2} \right)}, \quad Y_{\parallel}[0] = 0.
\]

To obtain the remaining distance to refract the translated wavefront outside of the lens we subtract the length \( Z_{\parallel}[0] \) from the thickness of the lens \( t \), and we get

\[
\mathcal{L}_0 = t - Z_{\parallel}[0] = \frac{(n_i - n_a) \left[ n_i^2 t + (n_i + n_i) S_H - n_i t \right] \sqrt{n_i^2 + (n_i^2 - n_a^2) S_H^2}}{n_i \left( n_i^2 + n_a \sqrt{n_i^2 + (n_i^2 - n_a^2) S_H^2} \right)},
\]

or alternatively, we can provide the distance between the plane face of the lens and the wavefront \((Z_{\parallel}[h], Y_{\parallel}[h])\) for arbitrary heights \( h \) obtaining

\[
\mathcal{L}_i = \frac{(n_i - n_a) \left[ n_i^2 t + (n_i + n_i) S_h - n_i t \right] \sqrt{n_i^2 + (n_i^2 - n_a^2) S_h^2}}{n_i \left( n_i^2 + n_a \sqrt{n_i^2 + (n_i^2 - n_a^2) S_h^2} \right)}.
\]  \hspace{1cm} (17)

In order to obtain the refracted wavefront outside of the aspheric lens, we consider that the \emph{Optical Path Length}, \( OPL = n_a \mathcal{L}_a + n_i \mathcal{L}_i \), is null, thus we have that \( \mathcal{L}_a = -(n_i/n_a) \mathcal{L}_i \), therefore the wavelets for the refracted wavefront outside of the lens are represented by

\[
(Z - t)^2 + (Y - y_i[h,t])^2 = \left( \frac{n_i}{n_a} \mathcal{L}_i \right)^2, \quad h \in [-H, H]
\]  \hspace{1cm} (18)
where \( y_i[h,t] \) is given from Eq. (14) evaluated at \( z = t \) and \( \mathcal{L}_i \) is given from Eq. (17) as is shown in Fig. 6(a). Following the steps explained above, we can obtain the zero-distance phase for the refracted wavefront outside of the aspheric lens. An alternative method is given as follows. We need to provide a length \( \mathcal{L}_a \) outside of the lens between the plane face of the lens at \((t, y_i[h,t])\) and an arbitrary length \( z \) propagating along the refracted rays at \((z, y_o[h,z])\), where \( y_o[h,z] \) represents the equation for the rays propagating outside of the lens with an arbitrary height \( h \), in such a way that by using Snell’s law, this can be written as

\[
y_o[h,z] = y_i[h,t] + \frac{(n_i^2 - n_a^2) (t-z) S'_h}{\sqrt{n_a^2 \left( n_a + \sqrt{n_i^2 + (n_i^2 - n_a^2) S'_h^2} \right)^2 - (n_i^2 - n_a^2)^2 S'_h^2}}.
\]  

(19)

so that the length \( \mathcal{L}_a \) is calculated as

\[
\mathcal{L}_a = \frac{n_a \left( n_a + \sqrt{n_i^2 + (n_i^2 - n_a^2) S'_h^2} \right) (t-z)}{\sqrt{n_a^2 \left( n_a + \sqrt{n_i^2 + (n_i^2 - n_a^2) S'_h^2} \right)^2 - (n_i^2 - n_a^2)^2 S'_h^2}}.
\]  

(20)

Assuming the condition \( \mathcal{L}_a = -(n_i/n_a) \mathcal{L}_i \) and substituting Eqs. (17) and (20) it is reduced to

\[
\frac{n_i^2 (n_a + \Delta) (z-t)}{\sqrt{n_a^2 (n_a + \Delta)^2 - (n_i^2 - n_a^2)^2 S'_h^2}} = \frac{n_i^2 t + (n_a + n_i S_h - n_i t) \Delta}{(n_i^2 + n_a \Delta)}.
\]  

(21)

where we have defined \( \Delta = \left( n_i^2 + (n_i^2 - n_a^2) S'_h^2 \right)^{1/2} \). By solving from Eq. (21) for \( z \) and substi-
tuting this value into Eq. (19) we obtain the zero-distance phase
\[
Z_{cp} = t + \frac{(n_t - n_a) \left\{ \frac{n_i^2 t + \{n_a + n_t\} S_h - n_t \Delta}{n_a^2 (n_a + \Delta) (n_i^2 + n_a \Delta)} \sqrt{\frac{n_a^2 (n_a + \Delta)^2 - (n_i^2 - n_a^2)^2 S_h^2}{n_i^2 (n_a + \Delta) (n_i^2 + n_a \Delta)}} \right\}}{n_a^2 (n_a + \Delta) (n_i^2 + n_a \Delta)},
\]
\[
Y_{cp} = h + \frac{(n_i^2 - n_a^2) \left\{ \frac{n_i^2 (n_a - n_t) t + n_a^3 (S_h - t) - \{n_a^2 + n_a n_t - n_i^2\} t + (n_i^2 - 2 n_a^2) S_h \Delta}{n_i^2 (n_a + \Delta) (n_i^2 + n_a \Delta)} \right\}}{n_a^2 (n_a + \Delta) (n_i^2 + n_a \Delta)},
\]
where the subscript \( cp \) means convex-plano. It is important to say that Eq. (22) provides the coordinates of the locus of points that parametrically represents the zero-distance phase front, which is produced by an aspheric convex-plano lens in a meridional plane as a function of \( h \) for a plane wavefront incident on the lens as is shown in Fig. 6(b). Substitution of Eq. (22) into Eq. (9) yields the exact wavefront which is propagated an arbitrary distance \( L \), according to
\[
Z_{\parallel cp} = Z_{cp} + \left[ \frac{[\partial Y_{cp}/\partial h]}{\sqrt{\left(\partial Y_{cp}/\partial h\right)^2 + \left(\partial Z_{cp}/\partial h\right)^2}} \right] L,
\]
\[
Y_{\parallel cp} = Y_{cp} - \left[ \frac{[\partial Z_{cp}/\partial h]}{\sqrt{\left(\partial Y_{cp}/\partial h\right)^2 + \left(\partial Z_{cp}/\partial h\right)^2}} \right] L,
\]
where for simplicity and brevity we have omitted writing this equation in its complete form. In Fig. 7(a) we have represented several wavefronts at different distances, and a zoom near the back focal length area is shown in Fig. 7(b).

For the ray tracing we have considered a lens with \( F/\# = 0.8 \), by using the following parameters in both plano-convex and convex-plano lens configurations: \( n_a = 1, n_t = 1.5111, R = 30.67 \text{mm}, \) diameter \( D = 75 \text{mm}, t = 35.5 \text{mm}, \) \( k = -0.905 \) and entrance aperture \( H = \pm D/2, \) whose aspheric coefficients are given in Table 1.

It is worth noting that since Eq. (23) defines the exact relation between aspheric terms and the
resultant wavefront (and caustic), it can contribute to both the optimization of the focal properties of aspheric lenses and to the solution of the converse problem: the design of refractive elements for the synthesis of arbitrary wavefronts. In other words, knowing the wavefront, we know the aspheric terms. This provides an alternative route to wavefront engineering where a plane wave needs to be transformed into a complex waveform and vice versa. For example, in microscopy, one can imagine using arbitrary aspheric terms in point spread function engineering [11] or aberration correction [12]. In applications of current interest, this formalism may be of more general interest where the form of a lens can be controlled in autofocus applications and even in refractive elements for terahertz applications [13].

Conclusions

We have obtained simple formulas for the wavefronts produced by positive convex-plano and plano-convex aspheric lenses having an arbitrary number of aspheric terms, considering a plane wavefront propagating along the optical axis. The shape of the wavefronts can be modified by changing the parameters of the lens and the distance where they are observed. When the condition of total internal reflexion is satisfied the wavelets do not contribute to the formation of the refracted wavefront, since these wavelets are unlimited therefore they are not perpendicular to the refracted rays. Upon inspection one can see that our wavefront coincides with the tangential solution according to [3]. We do not treat the sagittal plane that they does, but we could if needed extend our results by using differential geometry to obtain the wavefront for the sagittal plane. The net result is a simplification in the formalism that gives us the simple set of equations 10 and 23 in this manuscript that allow a direct calculation. We believe that this method for obtaining the wavefronts reported here are straightforward, giving a relationship between caustics and wavefronts.

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