Structured Multidimensional Information

*Representation Primitives, Location and Attribute(s)*
(Medical) Imaging

Energy, Information or Stimuli

Image Formation (Physics)

Sensors

Image Production (Technology)

Multidimensional Information Processing, Analysis
Medical Imaging

**Input stimuli**

- Light (coherent)
- EM-field
- Radiation: \( \nu \), x-ray, \( \alpha, \beta, \gamma \), e\(^{-}\), e\(^{+}\)
- Ultrasound
- Labelled probes
- Contrast agents
- Biochemical signals
- Electric signals
- Mechanical signals
- Cognitive activity

**DataBase of Atlases and Phantoms**

- Mapping & registration
- Inter-comparison
- Ground truth & reference
- DBs: anatomical, functional, Simulation

**Endoscopy, keratopography, thermography**

**Moiré-pattern imaging, polarography**

**Microscopies (confocal, multi-photon, ...**

**Radiography, Fluoroscopy, CT, axial, spiral**

**Contrast angiography, dynam. MR-angio.**

**NMRI, nuclear medicine, functional NMR**

**Scintigraphy, gamma-camera,...**

**TEM, SEM, synchrotron EM, PET, SPECT**

**Flow, tensor, parametric imaging**

**US mode A,b, Doppler, Time-of-flight**

**Diffusion / perfusion imaging**

**Impedance tomography, cardiography**

**Magnetoelectroencephalography map ... ...\ldots**

**Multidimensional Information**

- N-dimensional reconstruction
- Processing, Analysis
- Quantification, Visualization
- Interpretation
- Diagnosis
## Unstructured and Structured Information

<table>
<thead>
<tr>
<th>Category</th>
<th>Description</th>
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<tbody>
<tr>
<td><strong>Samples</strong></td>
<td>Set of attribute values (no order, no spatial or temporal structure; may be point samples in $N$ dimensions)</td>
</tr>
<tr>
<td><strong>Signals</strong></td>
<td>Ordered set of values (attribute), in one dimension (typically time or one spatial dimension): $f(t)$</td>
</tr>
<tr>
<td><strong>Images</strong></td>
<td>$N \times M$ Array of values (attribute), (typically two spatial dimensions): $f(x,y)$ ~ “bidimensional signals”</td>
</tr>
<tr>
<td><strong>Volumes &amp; Videos</strong></td>
<td>$N \times M \times L$ Array of values, (typically three spatial dimensions): $f(x,y,z)$ and sequences of frames $f(x,y,t)$</td>
</tr>
<tr>
<td><strong>Hyper-vols</strong></td>
<td>$N \times M \times L \times K$ array of values; e.g, volume sequence $f(x,y,z,t)$</td>
</tr>
</tbody>
</table>
$N$-dimensional local-structured sets in $M$ dimensions and boundaries of other manifolds (topology differences):

- Lifting of 2D images to 3D volumes (or $ND$ to $(N+1)D$)
- Boundaries of objects in $M$ dimensions:
  - Endpoints, sets of endpoints, hot points.
  - Contours (planar and embedded in $\mathbb{R}^M$).
  - Surface boundaries
- Parametric curves: $(x(s), y(s))$, $(x(s), y(s), z(s))$
- Time evolving sets (shape and topology changes):
  - Snakes: $(x(s), y(s), z(s))(t)$
  - Lightings (branching snakes)
  - Balloons and branching balloons
- Networks, branching curves, attributed graphs in $\mathbb{R}^M$
- Parametric surfaces, interfaces, waves, patches and membranes: $(x(r, s), y(r, s), z(r, s))$
- Hypersurfaces, branching manifolds and branes
- Mixed manifolds

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Structured Multidimensional Information

Representation primitive of discrete samples

Location: $x, y, z, t, \sigma,...$
(spatial) configuration

Orthogonal discrete grid

Associated Attribute(s) & Features:

Scalar value (intensity, depth, density)
Vector value: channels (color, optical flux)

...
Where is the “Pixel”? 

TV CRT

PC CRT

XO-1 LCD

LCD
*Euclidean Representation Primitives

- Points and nodes, **pixels**, voxels, hyper-voxels
- Edges, lines, arcs, arrows, flux-lines, planes
- Polygons, disks
- Facets (triangles, voxel-faces, polygons)
- Tetrahedra, spheres, polytopes, ellipsoids

Non Euclidean:
Geodesic pixels, spels, surfels and “patches”
*Other Representation Elements
(may have location and attributes)

Cellular complexes (vertices+edges +facets+voxels)

Texels (textural primitives), symbol sets, icons, images

Neighborhoods, cliques, structuring elements, kernels

Contours (iso-…), discrete boundaries, meshes, finite/volume element, solid boundaries

Medial Axis (regional-reps) = skeletons

Graphs

Smoothed Particles

PDFs (Point Spread Functions, LDFs, EDFs)

Polynomials, Splines, Basis Functions, Wavelets
Location: \( x, y, z, t, \sigma, \ldots \)

Geometry, (spatial) configuration, *domain*

Set of points, (cartesian) array, grid, lattice or tiling, irregular mesh/graph

- **Orthogonal discrete grid**
- **Honeycomb or hexagonal grid**
- **Offset-square grid**
- **Interlaced tiling**
- **Statistically regular tri-mesh**
- **Spheroidal polar coordinates**

Vertex-and-edge grid

\( (= \text{graph}) \)
Associated Attributes & Features:

**Scalar image** (e.g. intensity, density, opacity, transparency)
Binary attribute (boolean value, orientation, “up/down”)
Parametric image (e.g. range images = depth)

**Vector image** \((n\text{-channels})\), feature vector (color, optical flux, \(\alpha\)-blend,…)
Transform domain (Fourier, Cosine, Hermite, EDT, MAT,…)
Output result of detectors or operators
Labels (classification, contours,…)
Local Information (fractal dimens, texture, coherence, directionality,…)

**Field image** (distance, tensor = transformation, displacement, warp)
An image itself (per point or pixel)
Neighborhood relations, regional, hierarchical, connectivity, …
Meta-information (data structures) & pointers to information
Scalar and Vector attributes can be visualized through:

- Gray levels, colors and Look-Up-Tables (indexed color)
- Patterns, textures, symbols, icons and widgets
- Material properties: texture, transparency, shininess, etc.
- Animation, blinking, stretching, morphing
- Lifting from 2D to 3D, projections, shades, depth cues
- Image fusion techniques, color-channel (de)composition
- Optical techniques: Moiré, polarization, filters, phase-interference, holography
A 2-bit indexed color image. The color of each pixel is represented by a number; each number (the index) corresponds to a color in the color table (the palette).
In classical **halftone images**, the primitives are dots or squares of varying size and fixed intensity (Black or Color). An intensity is visualized by using a size proportional to gray-level (or color) intensities.
In modern halftones, the primitive is a B/W dot, with a quasi-random position; the attribute is gray-level and visualization is done by varying spatial-location statistics: dots agglomerate in function of intensity.
In “ASCII” art, the primitives are a set of B/W characters, with fixed location. The gray-level attribute is visualized by using sorted characters in function of their average gray-level by area unity.
Floyd-Steinberg dithering algorithm, based on error diffusion
A vector field may be a displacement field to model an image deformation: each pixel has a displacement vector associated \((\Delta x(x,y), \Delta y(x,y))\)

\[
\Delta x(x,y) \quad I(x,y) \quad I(x+\Delta x,y+\Delta y)
\]

\[
\Delta y(x,y)
\]

The displacement \(\Delta x, \Delta y\) changes at each point.
Scalar-field “deformations” (intensity-only)

Pixel-by-pixel addition (+) and product (⋅)

\[ T(I(x,y)) + \Delta I(x,y) \]

Transfer function \( u_{out} = T(u_{in}) \)

\[ T_{(x,y)} (I(x,y)) \cdot M(x,y) + \Delta I(x,y) \]

Position-dependent transfer function

Offset field (background)

Modulation field
Optic flow
(2D representation)

Elevation (°)

Azimuth (°)

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Filled vectorized contours (polygons)
A (very) simple manifold

\[ L: \mathbb{Z} \rightarrow \mathbb{Z}^2 \]

Discrete Polyline of \( N \) vertices:

\[ L = \{(x_s, y_s)\}_{s=0,\ldots,N-1} \]

Edge vectors:

\[ v_s = (x_s, y_s) - (x_{s-1}, y_{s-1}) \]

\( L \) is a (closed) contour of \( N-1 \) vertices if \( (x_0, y_0) = (x_{N-1}, y_{N-1}) \)

Region A (polygon) = solid/geometric interior of \( \partial A \). We call \( \partial A \) the boundary of \( A \).

\( C: \mathbb{R} \rightarrow \mathbb{R}^2 \) parametrical curve, \( C = (x(s), y(s)) \), \( s \) may be the arc of length

\( L \) may be a vectorized curve \( C \).

Set of pixels \( \{(x, y)\} \) fitting \( C = \text{rasterized curve} \).
cubic-spline interpolant

\( \left\{ a_s + b_s t + c_s t^2 + d_s t^3 \right\}_{s=0,...,N-1}, \quad t \in [0,1] \)
Spatial Discretization
Spacing / sample geometry

Atribute

\[ x, y, t, \text{ etc.} \]
Atribute(s) Quantization
Scale of \( L \) values (integers)
Digitization = Discretization AND Quantization
Continuous Appearance – Discrete Model

Atribute $(x, y)$

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“2.5D” Intensity Surface = (binary 3D object) → 2D Image (gray levels)
3D Surface = (binary 3D object)

2D Image (gray levels)
Iso-intensity sets representation

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Processing, Analisis & Visualization:

- **On representation primitives**
  - Format conversions, coding, compression, crypting
  - Re-sampling, texturing, meshing, finite element

- **On attributes and parameters**
  - Transfer functions, segmentation & labeling
  - Convolutions (average = special case)
  - Integer transforms (e.g., FFT)
  - Functions and arithmetic operations
  - Logic operations, predicate segmentation
  - Fusion
  - Re-sampling, Interpolation

- **On localization/coordinates**
  - Geometric transformations – Distortions/warps
  - Registration, image fusion
  - Re-sampling, zoom

- **On visualization attributes**

- **On attributed manifolds**
1. Scalar image (e.g. Intensity)
2. Parametric image (e.g. range images)
3. Features vector (color, optical flux,…)
4. Transform domain (Fourier, EDT, MAT,…)
5. Output of detectors or operators
6. Labels (classification, contours,…)
7. Local Information (fractal dimension, texture, coherence, directionality,…)
8. Field image (distance, tensor)
9. Neighborhood relations, regional, hierarchical, connectivity, ...
10. Meta-information (data structures)
11. Pointers to local information
12. Etcétera…
HERE: manifold examples

- Parametric curves, polylines, polygons (vectorized contours)
- Discrete oriented boundaries: Facet, voxel, triangle mesh, hamiltonian paths = (directed closed) graphs
- Interfaces, thin-plate splines, bilinear Coons patches
- Möebius strips, Klein bottle, Penrose triangle
- Branching structures: Medial Axis Transform and SKIZ, coraline manifold, mixed sets
- Topology properties